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DR. MARGARET MACDONALD

By RUTH SAW

MISS MARGARET MACDONALD died in St. Thomas's hospital on Saturday, January 7th. The time of waiting to go into hospital is naturally one of anxiety, but Miss Macdonald put her personal preoccupations on one side and spent this time in bringing all her professional duties up to date, even looking ahead. She had prepared the January number of ANALYSIS for press, set her examination papers for the Summer and drawn up the agenda for the next meeting of the Board of Philosophical Studies. This will surprise nobody who knew Miss Macdonald : she was admirable in her sense of responsibility and in the quiet efficiency with which she carried out any duties which she had undertaken.

Unfortunately, in creative work it is impossible to take time by the forelock in this way, and we shall miss the many illuminating things which Miss Macdonald still had to say on philosophical matters. In an orderly manner, she had dealt with a series of clearly defined problems and set out her reflections with neatness and precision in articles which were distinguished by their sanity, their determined hold on common sense, and by the elegance of their style. Some of her sayings are epigrammatic ; nobody who works on the problem of dreaming and waking will forget her dictum : " Just as waking up is not a method of discovery, so falling asleep is not falling into error." Her words were equally well chosen in discussion, and when we saw her pondering, head in hand, we knew we could expect a comment or question which would go straight to the point. Though temperamentally incapable of " speculation ", Miss Macdonald had a generous admiration for those of her colleagues who were given to this indulgence, and would listen with a comment here and there which helped along the flow of ideas she could not professionally countenance. Aesthetics, her chosen field of late years, has received a serious set-back by her death. It is very much to be hoped that we shall be able to see, in some form, the book on Aesthetics on which she had been working for some time. Her knowledge of the arts together with her sensibility and philosophical acumen must issue in a notable work.

Much as we shall miss Margaret Macdonald in philosophy,

the loss to her students and colleagues of a friend is even greater. She was unfailingly generous of time and help, and she was a delightful companion. Her cheerful pessimism and kindly cynicism sometimes issued in sayings whose wit took even her by surprise. Philosophical gatherings will not be the same without Margaret Macdonald, and we who are her contemporaries will sadly miss her for the rest of our lives.

SELF-DEFEATING PRONOUNCEMENTS¹

By DAVID S. SHWAYDER

1. Introduction

IN this article I propose to consider certain general features of the use of language which are especially relevant to the so-called logical and semantical paradoxes and to kindred puzzles. I should like, that is, to point out certain general facts about the use of language and by this means furnish a way of describing how we contrive to understand the use of certain inscriptions or utterances as paradoxical or as otherwise but similarly puzzling. I shall also recommend how we ought to set about solving puzzles of the kind considered, as they might be formulated in ordinary prose.

Let me say from the outset that what is to follow is not designed as a *solution* to the paradoxes and kindred puzzles. It is not a solution for a simple reason: These puzzles are of widely different varieties, and each demands its own analysis. In addition to the familiar "statement-paradoxes" which issue in formal contradictions—paradoxes like The Liar, Heterological and Russell's Paradox—there are other ostensible statements which engender bewilderment because they would seem to be inevitably true or inevitably false, and yet still not lead to formal contradiction or tautology. There are, for instance, cases where an utterance is, as we might say, *self-sustaining* or *self-reinforcing* to the vanishing point, e.g., "This is a statement", "This statement is true"; or again, "This sentence is English"; and still again, "Cogito" said whilst one is cogitating in any of the ways in which one may. Consider also the follow-

¹ Delivered before the 53rd annual meeting of the Western Division of the American Philosophical Association, East Lansing, Michigan, April 28, 1955.

ing : Let us assume the familiar convention for referring to an expression by exhibiting an instance of that type of expression within inverted-commas ; let us further assume that this is the *only* operative convention allowed. Then, a statement of the form, "The last word in this sentence is A", is false, no matter with what we might replace "A". Further, it is easy to manufacture puzzles of a similar sort which do not involve stating or which involve something more than stating. Consider, for example, a Jonsonian character named "Miss Named" ; and recall the familiar "No Signs Posted" posted as a sign. For purposes of convenience I shall take for examples only such pronouncements which some have described as "self-defeating". There are important differences between examples of self-defeating pronouncements, but there are also important similarities, some of which this paper aims to elicit.

I should like to draw an important distinction at the beginning. I wish, namely, to distinguish the paradoxes and kindred puzzles as they might be formulated either facetiously, or seriously in certain kinds of philosophical writings, from the puzzles as they might occur in certain more or less formalized languages or in mathematics.¹ That this is a valid distinction will, I hope, become apparent in the sequel. Granting that the distinction is valid, it is clear that what we may call the *same* paradox might appear in either guise. Nevertheless, the analysis required by a paradox or kindred puzzle as it arises in the one kind of situation will be different from the analysis required by the paradox as it arises in the other kind of situation. In this paper I consider the paradoxes and kindred puzzles *only* as they occur in ordinary prose, which is to say, in certain kinds of philosophical writings. (My analysis will be obviously inappropriate to the solution of the paradoxes in formalized languages ; it is similarly inappropriate for the mathematical logician to declare that the paradoxes prove that ordinary language is inconsistent.)

2. Sign and Statement

It is sometimes thought that all the puzzles of the kind here considered are, in some sense, due to self-reference. It is further thought that all self-referring linguistic activities are suspect and to be avoided. This is terribly vague and probably wrong. There are statements which are, in some sense or other, *self-referring*, but which we can easily understand, e.g., "This

¹ It is, I hope, clear that this distinction is not at all the same distinction as that drawn by Ramsey between "logical" and "semantical" paradoxes.

sentence contains five words". When faced with some such example, those who are suspicious of all self-referring statements, generally object to the "this". Others, who do understand the statement, reply that we can easily replace "this" with a proper name or a description and still have precisely the same statement. True; but such an adjustment is not necessary. Both parties fail to recognize an important distinction, and, consequently, fail to clarify adequately how "this" or any substitute works in cases similar to that in hand.

Observe how we might describe the present use of the sentence, "This sentence contains five words": We *form* the sentence, "This sentence contains five words", in order to make a *statement* about that very sentence which is *exhibited* as the subject of the statement. The standard uses of the words contained in the sentence are such that we readily appreciate what statement or statements might emerge upon the forming of the sentence in question. The important distinction is that between the linguistic tools we employ and what we employ them for. What I have uppermost in mind is the distinction, not between a particular sign and its use (as "Wittgenstein" is used to refer to Wittgenstein), but rather, the distinction between signs and concatenations of signs, and the different sorts of language activities towards the successful accomplishment of which we should normally use the signs in question. Drawn generally, the distinction covers all types of conventional symbols—signs, sounds, gestures and the like—and all kinds of linguistic activities—statement-making, promising, naming, swearing, and the like. It is easy to see that the distinction is a real one because we often make the *same* statement with different words; also, we use the same words to make different statements. (E.g., "I am writing". It is part of the *meaning* of "I" that it can occur in sentences which may be employed to make different, even contradictory, statements). It must not be thought that, say, a statement is just a sentence in correct use, for we often make statements with sequences of symbols not properly sentences (the answer "Yes") and often with no words at all (a nod of the head; occasionally, a discreet silence).

The question some will now ask is, "What is a statement?" This is a peculiar question and deserves a somewhat peculiar and misleading answer: A statement is no *thing* at all, it is not a "Proposition" nor a "Gedanke" nor an "Objektive". If anything, we must speak of "statement-making". (And observe that no one would speak of "making a proposition", at least

not in a sense relevant to philosophy). To make a statement is to *do* something of a classifiable kind; it is to perform an *act*—that is why we are responsible for our statements—and there is no statement apart from the act and apart from someone's having made the statement. I balk at broaching certain difficult questions here. These have particularly to do with the criteria used to identify and re-identify statements. But it should be clear, nonetheless, that there is nothing intrinsically spooky about a statement.¹

We may now say in what way "This sentence contains five words" is self-referring. The "this" refers to a certain pattern of words—it does not refer to a statement. Using *those* words, a statement does emerge, but the pattern of words might have been different and the same statement successfully have been made. We might, for instance, have said, 'The sentence, "This sentence contains five words," contains five words'. Only one last reservation: To make the statement that "This sentence contains five words" with that form of words would, under conditions of the kind envisaged, generally be *pointless*. If a pointless-statement is not a statement, then when used under conditions of the kind envisaged, the sentence is not appropriately employed. This matter will be resumed later.

3. Occasions, Conditions and Compensations

With these preliminaries completed, I now proceed to some general observations on the use of language.

The first observation is that when we speak about the *meaning* of a word (or sequence of words) we are usually speaking obliquely about the way in which the word is used.² This is not to be taken to imply that we can, even in theory, describe all the ways in which a word is or could be used. This we certainly can *not* do. What is important is that a word must have some use, and a word (for present purposes) has no *meaning* apart from that use. Now, to learn the meaning of a word—to learn the use of a word—is, among other things, to learn *when and where* to use the word. As part of this we observe that a

¹ We may refer to statements as well as make them, e.g. "Her statement was apt." But making (expressing) statements is not a species of referring; we can refer to a statement *only after* it has been made. See analysis of The Liar below.

² It is not always strictly correct to say that the meaning of a word *is its use*. We may sometimes ask for the *literal* meaning of an idiom, e.g., "good morning", and show by this that we do not want the use of that idiom, e.g., to greet someone. In this article, however we are primarily concerned with verbal activities and capacities as such. So I think that the only relevant way of giving the meaning of a word in the present context is to describe its use, say, to make a certain kind of statement. This use of "use" is different from another use of the word to be mentioned shortly.

word will be meaningfully employed only when used on an appropriate occasion. Again, it is not possible, even in theory, to delimit the range of occasions upon which a word will be appropriately or meaningfully used. It is important to recognize that certain occasions will be appropriate for the use of the word and others will not. For example, "yes" is a word used to yield assent, consent or permission, to make statements, swear oaths, add confirmation, etc., in response to certain questions. But "yes" is not meaningfully employed when used in response to such questions as "How many people are coming for dinner?" This means no more than that the kind of occasion adverted to is not one which would fit under any of the recognizable sorts of occasions upon which it would be meaningful to answer "yes". We *might*, of course, allow "yes" a jib ere, it might furnish a polite way of saying "Your question is impertinent".

All the above remarks apply in the first instance to sentences, but equally to words which can be used in lieu of sentences; secondarily, the remarks apply to words occurring in sentences or sentence-proxies. I say "secondarily" because of the important distinction touched on earlier. When we speak of the use of "yes" as above, we are using "use" in a different sense from when we speak of the use of "Wittgenstein". "Wittgenstein" is used to refer to a certain famous philosopher now dead; "yes" is used to make a statement. I wish to elucidate this distinction only very slightly by observing that a word could not be used say, to refer, except when, either alone or in the context of a sentence, it contributed towards the successful termination of some linguistic activity like making a statement, advertising or alluding to someone, questioning, ordering, etc. We might say that when "Wittgenstein" is used to refer, it is also being used either to make a statement, or to ask a question, or... But that way of speaking tends to obscure the important differences between the two uses of "use".

The next observation is that on a given occasion a verbal attempt will tend to miscarry unless certain conditions are satisfied. Again, it is not possible, even in theory, explicitly to delimit what these conditions will be on any given occasion. The relevant conditions are of widely differing kinds: semantical (proper-names must have been assigned a reference), grammatical, physiological and psychological (a person must not stutter too badly), sociological (one must use a language suitable to the society and nation of the listener); in certain

cases it is required that the conversationalists be prepared with roughly the same education, interests, etc. Strictly, there is no end to the list of conditions which *might* be relevant. Usually, however, most of the relevant conditions will automatically be satisfied if the words are used on an appropriate occasion. Learning language, like learning right from wrong, is mostly a matter of learning what counts as an exception, learning to spot the special case, the particular mistake.

The next observation is that it is not strictly correct to say that any one of these conditions is absolutely necessary. Any one may be questioned and any one may be substituted for by the context, the occasion, by common understanding and by all those things which go to make up the necessary adaptability and flexibility of language. Compensation comes about in all sorts of ways. We supplement sentences with gestures, we use italics, in the course of conversation we prepare ourselves for the eventuality that an ostensible proper-name has no reference; we often anticipate and speak for the stutterer and are able to do this because of what the context and common understanding provide. We come to understand and come to see that we do not understand in a thousand different ways.

Despite this, on any given occasion, certain conditions will be much more pressing than others, if the verbal attempt is to achieve success. We are inclined, speaking loosely, to say that these conditions are absolutely necessary, they must be satisfied if the verbal attempt is not to miscarry. Often, *in fact*, the verbal attempt will miscarry on account of the failure of certain conditions to be satisfied, leading, perhaps, to a complete breakdown in communication. At other times, we contrive to prevent the ordinary processes of correction and compensation which ordinarily accompany discourse. Such, we shall see, often happens in the case of the paradoxes and kindred puzzles.

What is secured by compensating, making allowance, correcting, supplementing the words and conditions for using words will be different on different occasions. Sometimes we contrive to allow a person to communicate what would normally be said with a certain form of words where normal conditions do not obtain. Such happens when the listener requests the meaning of a word previously unknown to him. Sometimes we contrive to understand that the words are *not* being used to say what they would normally be used to say, often because the occasion and the context prohibit the normal use, the sanction being irrelevance, contradiction, or sheer nonsense. Certain varieties of metaphor exemplify the point.

4. Application to the Case of Self-Defeating Pronouncements

I now apply the above observations to the case of the paradoxes and kindred puzzles, puzzles which arise from what I call "self-defeating pronouncements". It is now possible to say more clearly what is meant by a self-defeating pronouncement: An utterance or inscription is self-defeating if, were it successfully employed in some verbal attempt, it would also succeed in calling into question, denying, incriminating, violating, controverting, countermanding or blocking certain of those conditions which we are inclined to say are necessary if the verbal attempt is to succeed. We sometimes use language in this way so successfully that we find no way of retrieving ourselves from the puzzle. We get a paradox because one verbal attempt—that of incriminating, etc., some other verbal attempt—has apparently been successful; but the "other" verbal attempt turns out to be none other than the first. One wants now to say that the verbal attempt succeeds and now to say that it does not succeed, and one cannot make up his mind.

Why is this so? No specific answers can be given here, but certain general features of self-defeating situations may be remarked. First, when stating a paradox or a kindred puzzle, we generally make it clear that just that is what we intend to do (stating puzzles is one of the uses of language!), and so we carefully block any of the natural and obvious escapes, corrections, etc. Recognition of this fact explains why it is often so difficult to make the uninitiated aware of the puzzle. They do not understand the puzzle as a puzzle, or they request further information which is judiciously refused them; they make adjustments; they may hopelessly *understand* that the attempt in question does not apply to that attempt. We may say that such people have not yet learned the puzzle-making use of language. To teach them this, what one does is systematically to refuse them the licence for successfully using the words in question in any otherwise comprehensible way.

Second, we often *see* from a frustrated verbal attempt that it has been frustrated. The use of the words in question, as it were, *exhibits* this fact. This causes us to treat the words on view as if they *also state* or contravene the fact which is exhibited; but, in the interesting cases, to achieve this statement would imply that the verbal attempt has not been frustrated. Once it is understood that the same form of words may operate

in different capacities (to make a statement, as the object exhibited, etc.), we can understand that the form of words may serve in more than one or two or three or . . . capacities at one time. (Such, I think, is the force of the convention of quotation. We refer to the expression and at the same time exhibit it. One might say that we refer to the expression by exhibiting it.) This shows why we normally and naturally do avoid philosophers' puzzles over a certain form of words, which is used to say something about that form of words as it is exhibited in another capacity.

It is clear, then, that in certain instances the self-defeating situation is such that there *is* something to say; if so, then we certainly can say what there is to say, though not always in the way proposed. What we must never think is that the proposed way of expressing ourselves is the *only* way. There are also cases where we *fix it* so that there is nothing of the desired kind to say, and in such cases, of course, we cannot say it, though we can usually in such situations so interpret the words occurring that they do do something. We should be puzzled by this only if we thought that a sentence somehow by its very shape could be used to express only one or one kind of state-of-affairs.

I should like to illustrate these observations by considering the familiar paradox of The Liar, which for the moment we may put into the form "This statement is false". The paradox might arise because we are led to defeat effectively the possibility of expressing what we might wish to express by assuming that a sentence can be used to express only one state-of-affairs or one particular type of state-of-affairs. Why "This statement is false", as here used, is a form of words which, if we insist, will not do, is plain: One of the conditions for ever saying that a statement is false is that a statement has been made, just as one of the conditions for bestowing a name on someone is that there is someone who is to be named. "This" refers to no statement. That can be more readily appreciated if we substitute for the inscription in question the less elliptical sentence, "This sentence is being used to make a false statement". One of the things which makes "This statement is false" puzzling is that there is a perfectly proper sentence on view, and one cannot see why no statement emerges. The second formulation avoids some of the confusion by separating the sentence from the ostensible statement. But that is not enough, for both forms of words (if paradoxically understood!) are equally paradoxical.

Both are forms of words we understand. We have learned how all the separate words work and how they work in conjunction with each other, just as we have learned how the words work in "I name you Ludwig". We recognize that the form of words "This statement is false" can be used to make statements. But the attempt to use this form of words in the way desired fails because there is no statement which asserts, and persists in asserting that something not a statement is false. On other kinds of occasions when we use "This statement is false", where "this" refers to something obviously not a statement (a command, babbling), we are prepared to stand corrected. But here we earnestly endeavour not to prepare ourselves for such a contingency. ("If it's not a statement, what is it?") The paradox is testimony to our intransigence. Now I do not want to minimize the amount of mental exercise which might be required to discover that the employment of a certain expression fails to give a statement, and even more, why it so fails. One might, indeed, puzzle over the fact that we sometimes come to discover that a certain expression is nonsensical; but we do sometimes make such discoveries. Thus, to take an example similar to one of Wittgenstein¹ "This paper has gone for as many minutes too long as one of the 0's of the function ' $x^2 + 4x + 5$ '". If I had said, "This paper has gone on for as many minutes too long as the moon", most people hearing this would immediately recognize that I had said nothing at all; and even the most charitable would have wondered what I was getting at. The first sentence obscures the obscurity because I have extended my means of expression in what at first appears a fairly obvious way by mixing into my language, as it were, a bit of familiar, though special and technical apparatus; but actually it was not really immediately obvious how the new apparatus combines or fails to combine with the language.

Adopting the second formulation of The Liar, "This sentence is being used to make a false statement", we may also illustrate the error which arises when there is something to say, though we choose a singularly inappropriate way of saying what there is to say or we refuse to understand how it is being said, even if inappropriately. Recalling the argument of the previous paragraph, we might be inclined to say that "This sentence is being used to make a false statement" is true, since it expresses a fact, namely, that it is false that something not a statement is false. This interpretation obviously can be made to

¹ *Philosophical Investigations*, Part I, pars. 511 ff.

lead to paradox. The puzzle arises because there certainly is a fact on view,¹ and it is the kind of fact which might, *on occasion*, be best expressed by some such form of words as "This sentence is being used to make a false statement". On this particular occasion, if we wish a paradox, the form of words on view is the most obvious choice. True, if very careful, we could have used the form of words on view to state the fact on view. This would, however, be most inappropriate.² Whatever is expressed would have been better expressed with some form of words different from that on view, e.g., 'It is false that the sentence "This sentence is being used to make a false statement" is being used to make a statement, true or false'.

That he has been carried along by the momentum of his grammar is the only reasonable explanation why any one not a philosopher would be tempted to express the fact on view above with the words on view. Once this is understood, we can, of course, make sense of the whole matter, simply by making the necessary adjustments in applying rules, which is what we must do in any case, for rules must always be applied. We so apply the rules to allow the form of words to become equivocal in a very odd way. The sentence is now used to make a statement and now it is exhibited as the subject of the statement—but in no case is the sentence the statement.

5. Stating and Exhibiting

Hitherto I have been quite willing, even anxious, to allow that, if we say, "This statement is false" or "This sentence contains five words", then a perfectly decent statement might emerge. Yet, these alleged statements will so often be so singularly *pointless* that there is some reason for saying that they really are not statements at all. It is this perhaps which gives some credence to generalized objections to linguistic self-reference. The matter is, as I see it, once again, best approached in consideration of this notion of exhibiting. The contrast between exhibiting and stating sheds light not only on the paradoxes but also helps when we turn to examine the correctness of so-called "Picture Theories of Language" and

¹ "Fact on view" gives me a compendious way of saying that there is an object on view, such that we see that the object has certain properties or relations. It would then follow, were we to say that the object has those properties or relations, that we should make a true statement.

² It requires that we write the sentence over itself twice. First, to state falsely that the originally unsuccessfully employed inscription is false; second, to say truly that the second inscription gives a false statement.

when we attempt to elucidate the important differences between, e.g., "How it hurts" and "How it hurt".

It is sometimes convenient to utilize in language replicas of objects to which we might refer when making a statement, asking a question, etc. The familiar convention of inverted-commas affords, as we have already remarked, a systematic way of doing that. In spoken language onomatopoeic expressions also count as examples, though less systematically organized. It is quite obvious, however, that we cannot always reproduce in linguistic replica what we choose to talk about. But even more important is this point: Suppose we use a replica to make a statement about the object represented: Then we have a conveniently transparent referring expression, which is used in much the same way as is a proper-name, but not at all in the same way as is a sentence. Exhibiting itself, however, in the sense of holding up an example or a replica, or in that in which one displays objects or feelings, etc. ("etc." covers much that is very different), is comparable, not to referring, but to stating. There is, of course, an important difference: exhibiting is not necessarily a linguistic activity, though it can be exploited in our use of language.

Now, if I address a statement to someone when both of us are directly confronted with a situation that both of us recognize will make that statement true, then the statement is, or might be, pointless.¹

Now, very often, language is what enables us to exhibit a situation. To *express* one's pain with the words "I have a headache" is, if one is not simulating, acting, lying, etc., something like exhibiting that pain. The reason there is no doubting the truth of the statement that he has a pain is simply that no statement has been made. But of course, a statement will be warranted by what has been exhibited or expressed.

Now, what happens in the case of self-defeating and self-sustaining pronouncements is that we exhibit an object, perhaps a sentence, perhaps ourselves in a state of cogitation, and at the same time we attempt to make a statement, promise, etc., with the object exhibited or with some other object the meaningful use of which is somehow conditional upon the state of the

¹ A full discussion of this point would be long and hard, for the needed qualifications are elaborate. I surely do not want to disqualify, for example, cases of *identifying* a situation for what it is. But if the situation is perfectly clear, then the alleged statement will not be falsifiable. If in such circumstances someone were inclined to doubt, we could only ask him what he meant. To use Wittgenstein's word: We are faced with the *criteria* governing the utterance; this is the kind of situation where one might learn to use those words.

exhibited object. We get a pointless pronouncement or a conflict or something closely similar. Suppose we get a conflict, and suppose we choose not to resolve it ; then, what kind of conflict is it? The conflict is not a contradiction because we do not have two statements ; the conflict is one between an ostensible statement and exhibited conditions which preclude or tend to preclude the successful emergence of that statement.

6. Conclusion

Implicit in my analysis is that we do not *need*, when faced with an odd verbal pattern, to be bewildered. The paradoxes and kindred puzzles are artificial anomalies in ordinary language. This is reflected by the fact that we *do* use a sign "No Signs Posted" effectively to prohibit the placing of *other* signs in the immediate neighbourhood of the sign posted. We, in fact, do substitute for, replace, modify, adapt the conditions for successfully using a certain form of words. Perhaps we knew this all the time. If so, the only merit of this paper is to have drawn attention to the fact that we do and must use language in the way described and that a pronouncement can be genuinely self-defeating only because we make it so or demand that language be what it is not. If we do choose to make a pronouncement genuinely self-defeating that will often, if not always, be achieved by creating a conflict between what we pretend to do with our words and a situation thereby exhibited which excludes in any of many different ways the fulfilment of conditions upon which the statement is presumed to depend. We might call this use of language "paradoxing".

If I am right in all this, then we can say quite generally how we ought to set about resolving a paradox or kindred puzzle in ordinary language. We ought, first, to see how the form of words in question may and most naturally would be used and understood ; we ought, next, to say in what way such understanding is frustrated by the demand for a puzzle.

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ANALYTIC STATEMENTS AND MATHEMATICAL TRUTH

By G. B. KEENE

MATHEMATICAL truths, it is commonly said, are analytic. That is, they are true by definition. This view has become so commonplace that the grounds for it are usually taken for granted. However, to say that mathematical truths are analytic is to assign them to the same category as a statement such as :

(a) "All bachelors are unmarried men."

But while it is a straightforward matter to show how (a) could be regarded as an analytic statement, it is not quite so simple a matter to do the same for, say, " $2+2=4$ ". Leibniz, for instance, was mistaken, as Frege points out,¹ in thinking that particular numerical truths could be reduced to statements of identity by the definitions : " 2 " =df. " $1+1$ ", " 3 " =df. " $2+1$ " and so on. Yet Frege did not regard Leibniz as mistaken in supposing that both particular and general mathematical truths were analytic. For there are definitions and definitions. It is the purpose of this note to exhibit the full force of Frege's criticism of Leibniz' view.

The disagreement between them turns on the sense in which each claimed that mathematical truths were "analytic". They differ in their uses of this word because they differ as to the kinds of definitions to be admitted in showing that mathematical truths are analytic. Frege admits only logical definitions (that is definitions in which no terms occur other than logical constants and variables), whereas Leibniz admits non-logical definitions as well. Thus a statement is analytic for Frege, if, and only if, the grounds for it are logical definitions and the laws of logic alone. Leibniz, on the other hand, shows, by his proof of " $2+2=4$ " that, for him, a statement is analytic, if, and only if it is expressible as a law of logic by means of definitions alone whether logical or non-logical. Of these two senses, which we may call "logically analytic" and "analytic in general", respectively, the statements of the first sort are a subclass of those of the second. For, if a statement has only logical definitions and the laws of logic as its grounds (logically analytic) then it is expressible as a law of logic by means of definitions alone, whether logical or non-logical (analytic in general).

¹ Frege, *The Foundations of Arithmetic*, p. 7.

From the fact that the definitions given by Leibniz are mathematical definitions, it now follows that his argument, as it stands, can show at most only that particular numerical truths are analytic in general. In fact, however, Leibniz has not shown even this. For, as Frege points out, his step from " $2+2 = 2+(1+1)$ " to " $2+2 = (2+1)+1$ " requires the further assumption of the general associative law for addition. Hence Frege's aim to provide proofs for such general laws. For if the general laws can be shown to be logically analytic, then, given logical definitions of the individual numbers, the particular numerical truths will have been shown to be logically analytic also. In that case, mathematical truths, having been shown to be logically analytic, will, therefore, have been shown to be analytic in general. Nevertheless, it must be admitted in support of Leibniz that the general laws themselves might be held to be analytic in general. That is, that mathematical truths might, despite Frege, be held to be analytic in general without it being necessary to hold that they were logically analytic. If this thesis could be maintained, Leibniz' position would not be affected, in principle, by Frege's criticism. Leibniz himself, indeed, can be interpreted as maintaining it. For in some of his writings he goes to the extreme of regarding all truths whatever, as, potentially, analytic in general. On this view, every true statement could be reduced, by definitions (logical or non-logical) of its terms, to the logical truth, " $x = x$ ", providing we knew exactly enough the meaning of every word in our language. Nothing, however, is to be gained by the mere dogmatic assertion of such a thesis. For the crux of the disagreement between Frege and Leibniz is whether, with numerical *laws*, such an analysis can or cannot be achieved. Furthermore, even if it can be achieved, the further problem remains, whether mathematical truths are logically analytic as well as analytic in general.

If it can be shown that, given certain non-logical definitions, mathematical truths are analytic in general, it will follow that the whole of mathematics consists only in arbitrarily devised ways of re-expressing a law (or a number of laws) of logic. The ways of re-expression will be arbitrary since no criterion is specified by means of which to distinguish the acceptable from the unacceptable. (The criteria provided by Intuitionist and Empiricist theories of mathematics are irrelevant here, since mathematical truths are not, on those theories, analytic in any sense). This consequent trivialisation of mathematics is often

willingly accepted by those who hold that mathematical truths are, without further qualification, analytic. Frege's case against it, however, is a strong one. In the first place, he can dismiss the Leibnizian view that all truths are, potentially, analytic in general, as the onus of proof rests on Leibniz who fails to supply it. Secondly, his position can be supported by a more positive argument.

This argument can be expressed as follows. The central issue, here, is whether all mathematical truths are analytic in general. What has so far been established is that particular numerical truths cannot be shown to be analytic in general, unless the general laws of numbers can be shown to be. Now, in order to show any statement to be analytic in general, one of two mutually exclusive conditions must be shown to hold :

- (i) The direct symbolisation of its logical terms alone renders it a law of logic. (E.g., "You cannot both go and not go", i.e. $\sim(P \sim P)$).
- (ii) The direct symbolisation of its logical terms, together with suitable definitions of its non-logical terms renders it a law of logic. (E.g., "All bachelors are unmarried men").

In the case of a statement like " $x + (y + z) = (x + y) + z$ ", condition (i) is clearly inapplicable. For, its only logical term, " $=$ ", being already symbolised, the statement remains as it is and the condition is not fulfilled. Condition (ii) is, then, the only other alternative. Consider now, the applicability of condition (ii) to the two statements :

- (a) "All bachelors are unmarried men".
- (b) " $x + (y + z) = (x + y) + z$ ".

In case (a) the only possible suitable definition is the non-logical definition :

- (c) "bachelor" =df. "unmarried man"

The justification of (c) is, of course, the extent to which it is a part of accepted usage. In case (b), on the other hand, the whole problem turns on deciding what meanings to accept for its terms. All we can do in the face of this problem is to say, either that (b) itself is (or is part of) a definition-in-use of the term "+", or else, that an explicit definition of the form (c) above, can be given. But to fall back on definitions-in-use is to resort to the axiomatic method of laying down immediately acceptable postulates. This is equivalent to showing on what assumptions the truths of mathematics rest ; it shows nothing with respect to

the justification of those truths. Yet those who claim that mathematical truths are analytic, without further qualification, are generally claiming to be able to account for their truth. They cannot, therefore, substantiate this claim by holding that a statement such as (b) is a definition-in-use of “+”. The only other possibility is that explicit definitions of the form (c) above, can be given of the primitive terms of mathematics.

The problem is, then, what explicit meanings to accept for terms like “+”. Whatever definitions are accepted, they will either be logical or non-logical. But the only kind of non-logical definition that could possibly be relevant would be one in mathematical terms. Since this would entail the introduction of further mathematical primitives, by means of which to define the existing ones, our problem would merely be shifted back one stage, not solved. The definitions will, therefore, have to be logical definitions. However, to show a statement to be true by logical definition is not as straightforward a matter as showing it to be, simply, true by definition. Consider, for instance, the statement (a) above. This can be re-expressed as :

(a') "All bachelors, that is unmarried men, are unmarried men."

The reason why (a) can be re-expressed as (a') is, obviously, that the word "bachelor" and the phrase "unmarried man" are used (more or less) synonymously in the English language. Analogously, what we require is a statement (b') of which we can assert : "(b) can be re-expressed as (b')", (where (b') is a law of logic). But what then would justify this assertion? Clearly no facts about ordinary usage could do so, since (b) is a mathematical statement. As such, it is already independent of (in the sense of having rules of usage distinct from those of) ordinary language. It would, of course, be possible to re-express (b) as a law of logic by putting "∪" for "+". This would give us the law :

$$"x \cup (y \cup z) = (x \cup y) \cup z"$$

But, while we would thus have shown (b) to be analytic in general, we would not have provided any grounds for supposing that all mathematical truths are analytic in general. We cannot, so to speak, simply make mathematical truths analytic in general by juggling with symbols. What is required is not a chance assignment of logical meanings to the primitive terms of mathe-

matics, but just that assignment which will render all mathematical truths, simultaneously, laws of logic.

To show a statement to be analytic in general, is as we have seen, a matter of finding suitable definitions. But it is clear that even if the only definitions that could possibly be found are logical definitions, the statement can still be shown to be analytic in general, without thereby being shown to be logically analytic. This is the case with the above treatment of (b). To replace “+” by “ \cup ” in (b), transforms it into a provable statement. But we have not thereby shown that it (far less that every mathematical truth) has as its grounds logical definitions and the laws of logic alone. We have shown merely that if “+” can be defined as “ \cup ”, then (b) is a statement having purely logical grounds. Everything turns on whether “+” can be defined as “ \cup ”. In short, if we are to show that all truths in which the term “+” occurs, are analytic in general, then if the only possible definitions are logical definitions, the only possible method is logical proof. Frege’s search, therefore, for definitions which will permit the proof in logic of all mathematical truths, will, if it can succeed, solve the problem. For we shall then have the only possible justification for saying that (b) can be re-expressed as some statement (b’) which is a law of logic, namely, that every such statement is logically so, re-expressible because logically provable.

If this is the only possible justification, it follows that if all mathematical truths are analytic in general, it can only be because they are logically analytic. Furthermore, the view that they are logically analytic has to be supported by the detailed working out of a theory of the Frege-Russell kind. Since no theory of this sort has found universal acceptance among mathematical logicians, the view that mathematical truths are analytic, cannot simply be taken for granted. If, on the other hand, a satisfactory theory of this kind can be found, then mathematics will have to be described as a way of deriving more and more complex consequences from basic formalised logic, rather than as a more and more complex way of re-expressing the laws of that logic. It is on this point that the chief importance of Frege’s theory rests. For he was the first to attempt, not merely to make mathematical truths analytic, but to prove them to be so.

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SENTENCE AND STATEMENT

Prof. Quine on Mr. Strawson

By JASON XENAKIS

PROFESSOR QUINE, in his review-article of Mr. Strawson's *Introduction to Logical Theory*, says that Strawson in distinguishing in the way he does between sentence and statement "runs the risk" of "hypostatizing obscure entities [viz. 'statements'], akin perhaps to 'propositions' or 'meanings' or 'facts' or 'states of affairs' . . .".¹ But I don't think that Strawson runs the risk of "hypostatizing" anything.² I think, in fact, that Quine's distinguishing (following Frege and Carnap) between the use and the mention of an expression³ should make it quite natural for him to accept Strawson's distinction between statement and sentence. For in the last analysis Strawson's distinction is merely the result of a modification and specification of Quine's. If you strike out, as you may, the term "mention" from Quine's distinction you have Strawson's "background" distinction between an expression and its use. And if you specify this generic distinction by substituting, as you may, "sentence" for "expression" and "statement" for "the use of a (sentential) expression", you have Strawson's distinction between sentence and statement.

A statement, in Strawson's distinction, is a "sentence-put-to-work": it is what you do with a sentence just as driving nails is what you do with a hammer. Or, to use Aristotelian terminology, a sentence is a "potentiality"—something someone can or could do something with. A statement, on the other hand, is a performance. And just as driving nails involves an instrument (hammer) plus a user of it, so stating involves an "instrument" (sentence) plus a user of it (speaker, writer, etc.). There are of course important dissimilarities between a sentence and a tool but their similarities are I think enlightening.

If then I understand Strawson's distinction between sentence and statement, it won't do at all to say that for him statements are propositions or facts or states of affairs or meanings. Those

¹ "Mr. Strawson on Logical Theory," *Mind* LXII (1953), 440–1.

² My "defence" of Strawson is based on *Intr. to Log. Th.*, as well as on "On Referring" (*Mind* LIX 1950), "Truth" (*A.S.S.V.* XXIV 1950), "Particular and General" (*A.S.P.* 1953–4 LIV 256–60), and "Truth" *Analysis* IX 1949 (reprinted in M. Macdonald's *Philosophy and Analysis* 1954).

³ *Mathematical Logic*, ch. 4, *Methods of Logic* (1950), pp. 37 ff., and elsewhere.

who appeal to propositions say that (certain indicative) sentences "express" propositions, and sometime they mean by this that sentences "refer to" propositions and, via these, to "facts" or "states of affairs". But a statement for Strawson is not "expressed by" a sentence; rather it is the result of using a (suitable) sentence in a certain way or for a certain purpose, the fact-stating purpose. There aren't two "entities"—sentence and statement—such that one (the former) "expresses", "refers to" or "signifies" the other (the latter). It is less misleading to say that there is just one thing (the sentence) in two states—in the dictionary (idling) state, as it were, and in the state of being employed. Nor are statements meanings for Strawson. A statement for him *presupposes* a meaningful sentence, a grammatically and "logically" correct collocation of words: you wouldn't be making any statement at all if this condition were not fulfilled. And, finally, I have made it obvious, I trust, that Strawson's statements cannot be "facts" or "states of affairs" either.

Quine further says that Strawson's statements are "perhaps contents conveyed by sentences" (*Mind* LXII 440). But this is not so, if by "contents" Quine means (and what else could he mean?) specific speakers, dates, places, etc., or information of some sort. Take the sentence "I am at present in Athens" (1). If I were to write this sentence today in a letter to an acquaintance of mine, its "content" to him would be that Jason Xenakis is (was) in Athens (at a certain specific date) (2), or that Adonis Xenakis' brother is in Athens (3), and so on. But obviously nothing of the sort is "conveyed" by (1). For one thing, in (1) "I" is not used, so there is no person actually referred to, and "at present" is likewise not used, so there is no specific date involved. It is also clear, I take it, that no pronoun means any proper name, and similarly for all the other "egocentric" expressions. Also, of course, "Athens" could be "Athens-Ohio" if uttered in Ohio (U.S.A.), or "Athens-Georgia" if uttered in Georgia (U.S.A.), as well as "Athens-Greece". (2) or (3) or ..., is not the "content" of a sentence (of 1), but of a statement.

One may not like Strawson's talk of the *use of sentences*. One may say that sentences, as against words and phrases, do not, generally, come ready-made, so you can't really use them: what, generally, you can use (to make statements, pose questions, etc.) is words and phrases not sentences (though no doubt you may, *qua* grammarian, e.g., consider sentences). One might say, in other words, that the unit of use, if not of meaning as

well, is not sentence but word or phrase. This objection, however, even if sound, does not seem to affect Strawson's general distinction between an expression and its actual employment, and consequently his notion of statement, nor does the similar, though not quite identical, objection raised by Prof. Ryle and Mr. J. L. Evans, without however mentioning Strawson by name.¹

The distinction between sentence and statement, and the more generic one between an expression and its use, has helped Strawson disentangle meaning from reference and from truth-value, which were not distinguished or clearly distinguished in earlier theories. Strawson assigns meaning to expressions but reference and truth-value to the use of expressions, to statements not to sentences. (He doesn't, I trust, include in "truth" so-called logical truths.) This separation, in turn, has helped him "refute" Russell's account of definite descriptions. Now it is conceivable that Quine is prevented from accepting or understanding Strawson's notion of statement because (among other reasons) he still clings to Russell's Theory of Descriptions.

One word, before I conclude, about Quine's curious assertion that "meanings are hypostatized (as well as obscure) entities".² (Cf. N. Goodman's "renunciation of meanings".³) Perhaps Quine says this because in his opinion certain elucidations of meaning postulate "hypostatized entities". But this is far from establishing that *meanings* are such "entities", just as a bad description of a city or of a natural phenomenon has no tendency to show that the city or phenomenon is non-existent. The only other possibility I can think of is that for Quine the ordinary word (*sic*) "meaning" involves, or its use commits one to, the postulation of "hypostatized entities" (as presumably "ghost" does). But so far as I know he hasn't proved this and, what is more, I don't think he can.⁴ It is only when our elucidations of meaning are misguided, or when they are based

¹ "Ordinary Language" *Philos. Rev.* LXII (1953) 178 ff., "On Meaning and Verification" *Mind* LXII (1953) 8-9, respectively. Cf. Strawson's reply to Ryle ("Critical Notice on L. Wittgenstein's *Philosophical Investigations*," *Mind* LXIII 73n). I wonder, incidentally, whether Strawson is aware of the fact that in his distinction statement corresponds not quite to sentence, but rather to a particular kind of sentence; e.g. it does not correspond to imperative sentences.

² *Mind* LXII 440-1, also: "On What There Is" *Rev. of Met.* II 31 (repr. in L. Linsky's *Semant. and Phil. of Lang.*, A.S.S.V. XXV, Quine's *From a Log. Point of View*), "Two Dogmas of Empiricism" *Philos. Rev.* LX 22 f (repr. in *From a L.P.V.*), and "Semantics and Abstract Objects" *Proc. of Amer. Acad. of Arts and Sci.* vol. 80, p. 91.

³ *Phil. and Anal.* (ed. M. Macdonald) p. 54, occurring in his 1954 "Foreword" to his reprinting "On Likeness of Meaning" (*Anal.* 1949) and "On Some Differences about Meaning" (*ib.* 1953).

⁴ Cf. *Méthodes* nos. 24, 25-6, *Theoria* XXI.

on misuses, and particularly on the philosophers' misuses, of "meaning", "to mean", "meaningful", etc., that we come up against strange results. Would Professors Quine and Goodman go on "renouncing meanings" or regarding them as "hypostatized entities" if they were told that the meaning of a word is not, as a matter of fact, some "entity", that it is not, e.g., a mental image, but the way or ways the word is used by those who know how to use it or, which is saying the same thing, the way or ways the word ought to be used by those who don't know or are learning how to use it? It would seem that an affirmative answer to this question would commit them to holding, among other things, that there is no such thing as teaching the use of a word; for doing this is precisely explaining the meaning of a word.

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MR. STROLL ON BELIEF SENTENCES

By RICHARD WILLIS

IN his article "A Problem concerning the Analysis of Belief Sentences" (*ANALYSIS*, October 1953) Mr. Stroll puts forward the theory that where "*p*" is a meaningless sentence "*I believe that p*" is meaningful but false. He says that it is begging the question to assume that "*p*" is a propositional variable in this sentence. This is quite correct since the question to be answered is whether or not "*p*" can be meaningless (*i.e.* a *mere* sentence) without also making "*I believe that p*" meaningless. I think, however, that Mr. Stroll is quite mistaken, and that "*p*" can be shown to be a propositional variable.

Consider the sentences :

- (1) I said "*p*".
- (2) I was in doubt about the meaning of "*p*".
- (3) I translated "*p*".

In each case "*p*" is a sentential variable. In each case (a) a strictly synonymous translation of the sentence would have to retain "*p*" in the original language, and (b) a different sentence substituted for "*p*" would alter the meaning of the whole, even if it was synonymous with "*p*".

Incidentally, there are logical differences between these three sentences. In (1) "*p*" can be meaningless without making any logical difference. In (2), if "*p*" is meaningless the whole cannot be used to make a (true or false) statement; but it will not itself be meaningless, since "*p*" may get a meaning at a later date, and may have had a meaning at an earlier date. (Cf. the well-known effect of the creation and abolition of French monarchy on sentences about the king of France.) In (3) "*p*" may be nonsense, but the words composing "*p*" must be meaningful. Thus it is logically possible to translate "the ideas have an independent subsistence", because, even if the sentence means nothing, it is composed of meaningful words. (In the German version of Jabberwocky, beginning "Es brillig war . . .", the word "brillig" can hardly be called a translation.)

However, whatever their differences, these three sentences are alike in the way in which they differ from "I believe that *p*". In such a sentence (*a*) when it is translated, "*p*" too must be translated, and (*b*) any synonymous substitution for "*p*" will not alter the meaning of the whole. Thus (*a*) "I believe Sokrates ist tot" is an intolerable linguistic jumble, and (*b*) no circumstances are conceivable in which some one, who could truthfully say "I believe Socrates is dead", could not also truthfully say "I believe Socrates is deceased".

Consider moreover the sentences :

- A1. I believe I am ill.
- A2. He believes he is ill.
- A3. You believed you were ill.

These can all be used (in different circumstances and by different speakers) to talk about the same person, the same belief, and the same occasion. That is, they can all be used to make the same statement. Now if Mr. Stroll is right, they must all mention the same sentence; since, if we believe sentences rather than statements, the same belief must be belief in the same sentence. This entails that the clauses "I am ill", "he is ill", and "you were ill" all name the same sentence. This seems fantastic. What sentence could they name? One would have thought that apart from demonstrative phrases such as "this sentence", "the following sentence", etc., the only way of mentioning a sentence was to write it in inverted commas (or to speak it with suitable intonation). Contrast with A1, A2, and A3 :

- B1. I say "I am ill".
- B2. He says "I am ill".
- B3. You said "I am ill".

And contrast with these sentences :

- C1. I say I am ill.
- C2. He says he is ill.
- C3. You said you were ill.

Notice that no two members of the group B1, B2, B3, C1, C2, and C3, are identical in meaning ; whereas, if we could attach any meaning to the queer form of words "I believe 'I am ill'" , it would surely have to be synonymous with "I believe I am ill".

Mr. Stroll argues : "When ' p ' is meaningless, it is impossible for me to believe ' p ', since I cannot believe a sentence which has no meaningful content. When I say therefore, 'I believe that p ' , I am making an assertion about my state of mind which is mistaken. I think that I am having a belief (assuming, of course, that I am not lying), but in fact I am not having such a belief. According to this interpretation, therefore, the belief sentence 'I believe that p ' is significant but false." In what sense is Mr. Stroll using the word "impossible"? He appears to be speaking of psychological impossibility. And yet the impossibility of believing nonsense can be known without any reference to psychological laws. The impossibility must be logical. Surely there is no answer to the question : "What would it be like if we were so constituted that we could believe nonsense?" If, therefore, " p " is a meaningless sentence, "I believe that p " cannot be significant.

I conclude that in "I believe that p " " p " must be a propositional variable. Therefore, if the subsentence is meaningless, the whole sentence "Santayana believed that the ideas have an independent subsistence" is also meaningless. However, it would be unduly pedantic to criticise a biographer or commentator for saying this, provided Santayana used the sentence, and believed that in doing so he was stating a fact. It would be pedantic to call such a sentence nonsense ; it would be completely wrong however to call it, with Mr. Stroll, false.

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